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Periodic orbit theory for resonant tunneling diodes: Comparison with quantum and experimental results

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We investigate whether the quantal and experimental amplitudes of current oscillations of resonant tunneling diodes in tilted fields are obtainable from periodic orbit (PO) theories by considering recently proposed PO approaches. We show that accurate amplitude and frequency shifts for the current oscillations (typically to within a few %) can be obtained from a simple analytical formula both in the stable (torus-quantization) limit and the unstable regimes of the experiments that are dominated by isolated PO's. But we find that the PO approach does not describe quantitatively the dynamically interesting intermediate experimental regimes that appear to be dominated by contributions from complex orbits and multiple nonisolated PO's. We conclude that these regimes will not easily be described by the usual PO approach, even with simple normal forms. [S1063-651X(98)51609-1]

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Mesoscopic systems have been extensively investigated as probes of ''quantum chaos'' in real systems. Among these, the resonant tunneling diode (RTD) in tilted fields, first introduced in 1994 [1], has attracted much attention because of the diversity of observed effects it exhibits, which have been attributed to periodic orbits and ''soft chaos.'' For instance, it has been used as an experimental probe of spectral fluctuations due to unstable periodic orbits [1], quantum scarring [2–4], bifurcations [5–8], ''ghosts'' [9,10], and the torus-quantization regime [10]. All of these effects manifest themselves through observed oscillations in the tunneling current of varying amplitude and frequency.

The well-known Gutzwiller trace formula (GTF) [11] is a powerful tool in the quantization of chaotic systems. It relates the frequencies and amplitudes of oscillations in the *density of states* to the actions and stabilities of classical periodic orbits (PO's) in a simple analytic formula. However, to date we have no equivalent formula to describe the corresponding oscillations in the *tunneling current*. Hence, some interpretations of the experiments, for instance, whether bifurcating orbits are or are not seen, remain controversial [12,13,4]. In [14], a semiclassical treatment of the current was presented, evaluating the Bardeen tunneling matrix element within the Wigner phase-space representation.

We show here that a simple analytical formula developed by Bogomolny and Rouben [15], as well as the approach of [14], can quantitatively yield the *amplitudes* of contributions from isolated periodic orbits—stable or unstable. This represents a demonstration that the amplitudes of the current oscillations may be quantitatively described by a periodic orbit expansion. All regimes considered here and in Ref. [14] involved PO's starting with zero lateral momentum. In this case (assuming the lowest Landau state for the initial state) we found here that the integral (5) of Ref. [14] reduces to the analytical formula derived in Ref. [15]. This implies that the results of Refs. [14] and [15] are indistinguishable. The approach of [14] provides more flexibility, allowing, for instance, excited initial states (which are not required in our calculation). However, the advantage of the formula from [15] lies in its simplicity—it is as easy to evaluate as the GTF—and in the physical insight that the analytical expression provides. For instance, it exposes a *shift* between the frequency of the Gutzwiller density of states oscillations and the current oscillations. This shift is small ($<1\%$) but the resolution of the quantum scaled calculations easily exceeds this. The comparison carried out here confirms that this shift is indeed easily detectable and accurately predicted in terms of classical quantities. Also, the formula reproduces the simple model describing the torus-quantization effects that we proposed previously [10], in agreement with quantal calculations and observed experimental features.

Here we have applied a rigorous test to this formula, since we have compared it with a broad range of accurate amplitudes obtained from a scaled quantum spectrum [9,10]. We also compare it with the extensive set of experimental data obtained at Bell Laboratories [6].

Many of the most interesting experimental features, such as period doublings, occur in an intermediate regime, characterized by contributions from multiple nonisolated orbits and complex PO's (ghosts). Here we find that agreement in this regime is qualitative: both the formula and the approach of [14] yield the rough range of period-doubling regions, but the amplitudes are in poor agreement with quantal results and experiment. In particular, in the two ghost regions we identify, we cannot account for the amplitude of the current oscillations, even with normal form corrections. The strength and persistence of these contributions remain one of the most puzzling features of these experiments since in general, ghosts are strongly damped away from the tangent bifurcation in which they appear.

We briefly recall the RTD model [1]. In essence, the physical picture is as follows: an electric field F (along x) and a magnetic field B in the x-z plane (at tilt angle θ to the x axis) are applied to a double barrier quantum well. Electrons in a two-dimensional electron gas (2DEG) accumulate

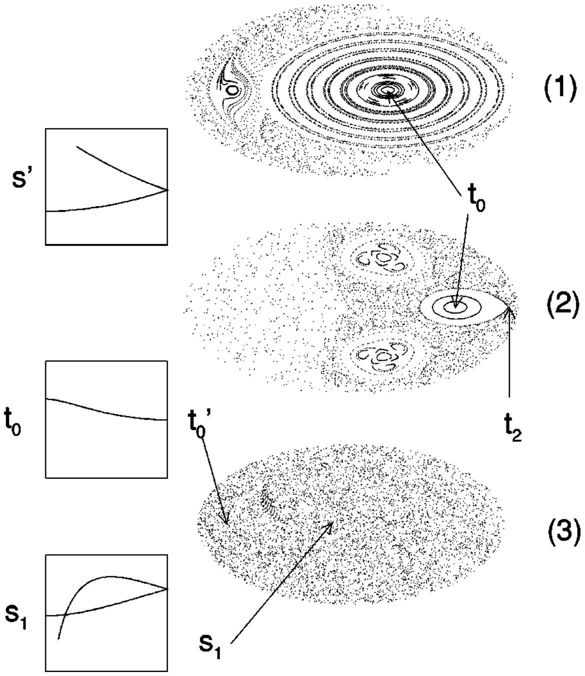

FIG. 1. Shape in the x-z plane of the main PO's t_0 (period-one), S' and S_1 (period-two). The SOS's illustrate, for $\theta=11°$, three generic dynamical regimes typical of $\theta=10$–$30°$: (1) $\epsilon=20\,000$ atomic units, large stable island; (2) $\epsilon=7000$ atomic units, intermediate regime. t_0 is about to undergo a tangent bifurcation near the edge of the SOS, which will remove the real PO. This regime is characterized by contributions from nonisolated PO's or complex orbits; (3) $\epsilon=3000$ atomic units, unstable regime. The new t_0' PO has reappeared on the far side of the SOS. A strong period-two signal from the isolated unstable PO S_1, which occupies the central region most accessible to the tunneling electrons, is seen in the experiments.

at the first barrier and tunnel through both barriers, giving rise to a tunneling current I. In the process they probe the classical trajectories—regular or chaotic—arising from specular reflection at the barrier walls. The current oscillates as a function of applied voltage V. After rescaling with respect to B, the dynamics depends only on the parameter $\epsilon=V/LB^2$, at given θ and ratio of injection energy to voltage ($R=E/V\sim0.15$ for the Bell Laboratories experiments). Regular behavior occurs at high ϵ ($\epsilon\sim20\,000$) (in atomic units), chaotic behavior occurs at low ϵ ($\epsilon\sim1000$). The well width is $L=1200$ Å. As we study small angles ($\theta\leqslant27°$), we neglect the shift $\delta z\sim d\tan\theta$ due to the mean distance d between the 2DEG and the left inner barrier ($x=0$). At $\theta=0°$, the current consists of pure period-one oscillations, of amplitude independent of ϵ, associated with a straight line PO (t_0), bouncing alternately between walls. This is our reference current amplitude I_0 and we normalize all amplitudes (semiclassical, quantal, and experimental) to I_0.

As θ increases t_0 is no longer a straight line but continues to dominate the period-one oscillations. Due to the relatively short coherence time τ (~0.1 ps) [1], just four of the shortest PO's (t_0, its second traversal $2t_0$, and the period-two PO's S_1 and S') account for all experimental features studied here. Their shape is shown in Fig. 1. Nevertheless, their dynamical behavior is far from simple. In [1] it was observed that these classical PO's appear and disappear abruptly. t_0 undergoes a series of tangent bifurcations, where it ceases to exist as a real PO but leaves a complex ghost. Subsequently, below the tangent bifurcation, a new similar looking PO t_0' reappears from the opposite side of the surface of section (SOS) and restabilizes. S' appears abruptly at the discontinuity in the potential between a barrier and the energy surface due to the magnetic confinement (a "cusp" bifurcation [8]). It disappears subsequently in a tangent bifurcation also leaving a ghost.

Despite these intricacies, we can identify in the experiments three generic dynamical regimes. These are illustrated in the surfaces of section in Fig. 1. They are: (1) $\epsilon\gtrsim20\,000$, the torus regime; the large stable island of t_0 yields a period-one current. The current shows "jumps" associated with torus quantization; (2) $20\,000\gtrsim\epsilon\gtrsim3000$, intermediate regime, with contributions from multiple nonisolated PO's or complex orbits; (3) $\epsilon\sim3000$–1000, unstable regime. We can identify contributions from unstable, isolated PO's such as S_1 (the period-two oscillation identified in the original Nottingham experiments [1]). We show below that the new semiclassical formula gives excellent results in (1) and (3) but rather poor results in the intermediate regime.

The theoretical scaled current (neglecting experimental broadening due to incoherent processes) is a density of states weighted by a tunneling matrix element: $I(N)=\Sigma_i W_i\delta(N-N_i)$.

The experimental range ($V=0.1$–1.1 V) corresponds to $N\sim12$–43, which gives an average $N\sim\hbar^{-1}\sim28$, corresponding to $V=0.5$ V. In fact, N is a rescaled magnetic field [9,10]: $N=BL\sqrt{2mL\epsilon(R+1/2)}/\pi$. In our calculations we used the Bardeen matrix element [16] form for the tunneling probability. Then, as explained in [15], one can reexpress the matrix element in terms of energy Green's functions and use their semiclassical expansion over classical paths. We consider the initial state describing the electrons prior to tunneling to be the lowest Landau state: $\phi_0(z)=\sqrt{B\cos\theta/\pi}\exp(-B\cos\theta z^2/2)$. Then the tunneling current is given by

$$I(B)\propto\mathrm{Re}\int dz\int dz'\times\sum_{cl(z\to z')}m_{12}^{-1/2}e^{iS(z,z')}e^{-B\cos\theta(z^2+z'^2)/2},$$

where $m_{12}=\partial z/(\partial p_{z_0})$.

The integrals were evaluated analytically by stationary phase with the condition $\partial S/\partial z=\partial S/\partial z'=0$. This condition implies that only PO's starting with null momentum $p_z=0$ contribute. The resulting contribution to the normalized current $I_{norm}(B)=I(B)/I_0$ for a given periodic orbit is approximated by

$$I_{norm}(B)=\mathrm{Re}\frac{e^{iB(\tilde{S}+\Delta\tilde{S})+i\mu\pi/2-B\xi}}{\sqrt{-\cos\theta m_{12}+m_{21}/\cos\theta+2im_{11}}},$$

$$\Delta\tilde{S}=\cos\theta z_0^2\gamma/(1+\gamma^2);\quad \xi=\cos\theta z_0^2\gamma^2/(1+\gamma^2),$$

$$\gamma=\frac{m_{11}-1}{\cos\theta m_{12}},$$

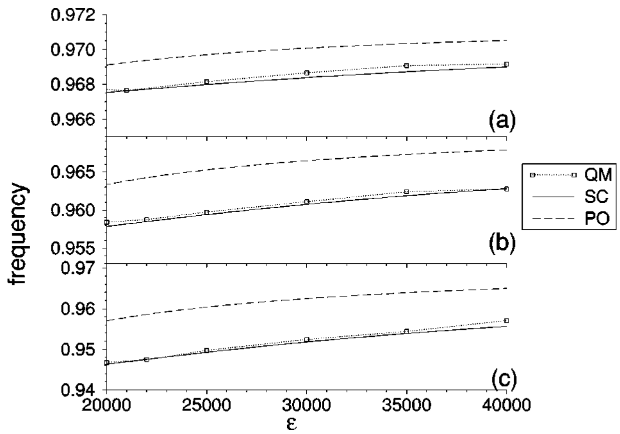

FIG. 2. Frequency of the period-one oscillation at (a) $\theta=11°$, (b) 20°, and (c) 27°. Squares: quantum frequency (QM); dashed line: action of the t_0 (PO); solid line: semiclassical frequency (SC), which is shifted relative to the action of the classical PO, but is in excellent agreement with the quantum results.

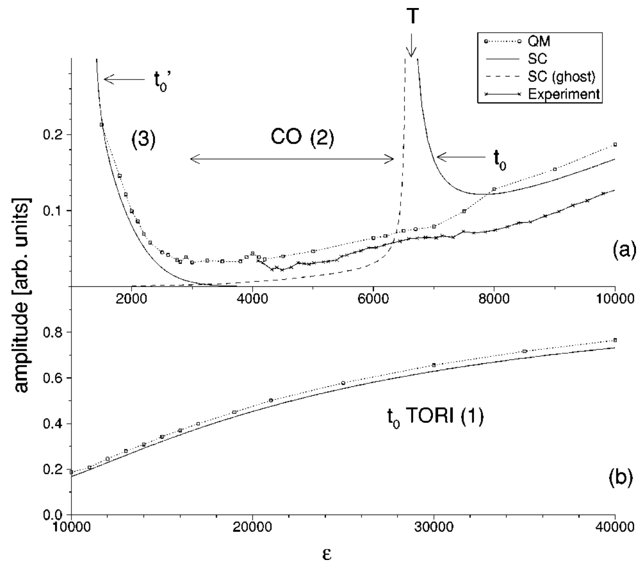

FIG. 3. Quantal (QM), semiclassical (SC), and experimental amplitudes for period-one oscillations at $\theta=11°$ for (a) regimes (2) and (3) with tangent bifurcation (T), complex orbit (CO); (b) torus regime (1). The semiclassical formula gives excellent results where the PO is isolated, i.e., for $\epsilon>8000$ atomic units (t_0) and $\epsilon<2000$ atomic units (t_0'), but gives poor results near the bifurcation and for $\epsilon=6500$–3000 in the CO region, even including the ghost PO. Reliable experimental period-one amplitudes are only obtainable at 0.5 V from $\epsilon=4000$ up to the start of a period-doubling region at $\epsilon\sim9000$.

where μ is a Maslov index, $\tilde{S}$ and m_{ij} are the *scaled* action and element of the classical monodromy matrix of the PO, with the starting position ($x=0$, $z=z_0$). The semiclassical theory predicts that the frequency of the current oscillations is shifted relative to the scaled action $\tilde{S}$ by $\Delta\tilde{S}$.

We show in Fig. 2 the scaled action of t_0, the semiclassical frequency $\tilde{S}+\Delta\tilde{S}$, and the quantum frequency obtained by Fourier transform. The shift of frequency is $\sim1\%$ at most, but clearly we can see that the shifted frequency is in excellent agreement with the quantal results.

In [10] we explained how one may extract experimental PO amplitudes in the case where the current has just a pure period-one or period-two oscillation, by removing the smooth nonoscillatory component. This is only possible in a restricted range of the experiments. Also, to compare with theory we must consider two factors: (i) the experimental features are displaced to a lower voltage relative to the theoretical value $V=FL$; (ii) incoherent processes damp each PO contribution by a factor $e^{-T/\tau}$, where T is the period of the PO. For the voltage displacement, we found that all PO features appearing at $V=0.5$ V in the calculated spectra are systematically displaced to a voltage 30% smaller in the experiment. For example, in [9], where characteristic line profiles were correlated with different dynamical regimes, we showed that the distinctive spectral signature of a bifurcation, seen in the quantal spectra at $\epsilon=13\,000$, is seen in the experiment at $\epsilon=10\,000$. Much of this voltage shift is accounted for by the voltage dependence of the effective mass [10]. Hence, we took $\epsilon\to1.3\times\epsilon$ for the experiments, for all angles and all values of ϵ. Then we found that, for all angles $\theta=11$–27°, the position of the theoretical and experimental period-doubling maxima are in good agreement.

For the damping we find that the *amplitudes* of the maximum period-doubling current would coincide if we chose τ in the range 0.10–0.12 ps for a given angle. This is remarkably consistent with the expected $\tau\sim0.1$ ps suggested in [1]. We chose a representative value $\tau=0.11$ ps for all of the experimental amplitudes. In effect, period-two amplitudes are damped by incoherent processes by about an order of magnitude relative to I_0.

We show in Fig. 3 the amplitudes of the experimental, quantal, and semiclassical period-one current at $\theta=11°$ from the torus regime through to the unstable regime. For high ϵ [regime (1)] the semiclassical current in Fig. 3(b) is due to the stable t_0 and its tori, and the agreement with the quantal calculation is excellent. However, the most interesting dynamical region is at $\epsilon=3000$–6000, in which there is a significant quantal/experimental current but no real t_0 PO (t_0 disappears in a tangent bifurcation at $\epsilon=6500$, and reappears at $\epsilon=4300$; between 4300–3000 the new t_0 is not easily accessible to the tunneling electrons, so its contribution is negligible). Here [regime (2)], the semiclassical result is poor even when we include the ghost complex PO. Even with a cubic normal form, which we do not present here, agreement remains poor. Quantitative agreement is once again good when the ''reborn'' isolated real t_0' orbit dominates the current for $\epsilon<2500$ [regime(3)].

In Fig. 4 we show a comparison between experimental, quantal, and semiclassical amplitudes of the period-two current at $\theta=11°$, 20°, and 27°. We were unable to read reliable experimental period-two amplitudes for $\theta=11°$, since a strong period-one beat is also present. At $\theta=20°$ and 27°, the quantal calculations and the experiments are in very good agreement. As expected in the large stable island regime $\epsilon>25\,000$ [regime (1)], agreement between the semiclassics and the quantum is excellent. This is also the case for $\epsilon<3000$ [regime (3)]. Here, the isolated unstable PO S_1 describes the current very well.

However, in the intermediate regime (2), the quantum current requires a coherent superposition of the nonisolated PO's $2t_0$ and S'. A straightforward sum (allowing for their

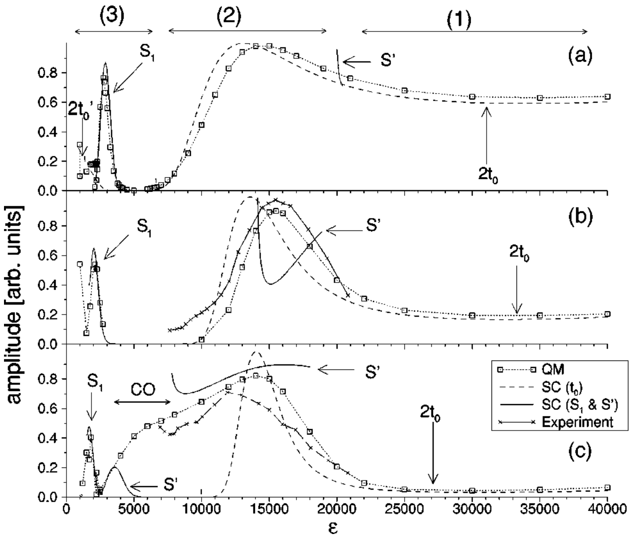

FIG. 4. Quantal (QM), semiclassical (SC), and experimental amplitudes for the period-two current at (a) $\theta = 11°$, (b) $\theta = 20°$, and (c) $\theta = 27°$. As shown previously, we have three regimes. (1) stable regime and (3) unstable isolated PO regime, where the semiclassical formula is good. In regime (2), by contrast, we have the nonisolated contributions of $2t_0$ and S', which cannot be added straightforwardly. Hence, we simply show the individual contributions. Here agreement between the formula and quantum calculations/experiment is poor. S' appears abruptly at a cusp bifurcation ($\epsilon \sim 18\,000$ atomic units) and disappears in a tangent bifurcation at lower ϵ, below which the experiment shows a slowly decaying ''plateau'' due to a complex orbit (CO).

phases) yields poor results. $2t_0$ and S' have nearly identical actions, unresolvable in the quantum Fourier transform spectroscopy. The $2t_0$ contribution comes from Miller tori localized on the large stable island and, due to the moderate values of $\hbar$, substantially beyond the island boundary. A phase-space analysis with Wigner and Husimi functions shows that the S' scars are mixed in with the outer tori of the t_0 island. Hence, both their action and phase-space localization coincide. At 11°, however, the contribution of S' is small and occupies a narrow range in ϵ. In this case the semiclassical amplitudes are quite good. This is not the case at 27°. Both the individual island ($2t_0$) and S' contributions are significant between $\epsilon = 20\,000$–$12\,000$, and there is no agreement with the quantal results. For $\epsilon < 8000$ the $2t_0$ contribution is negligible and S' has disappeared into the complex plane in a tangent bifurcation. Even including the S' ghost, we were unable to obtain quantitative agreement in this complex orbit (CO) region spanning $\epsilon = 8000$–5000. We note that the slowly declining plateau seen quantally and in the experiment is a surprising and unexpected feature, since ghosts should be exponentially suppressed as the imaginary component of the action grows.

We conclude that for $p_z = 0$ PO's [15] and [14] yield equally good results in the torus and unstable regimes. Both encounter the same difficulties in regime (2). We note that although [15] requires the $p_z = 0$ selection rule, [14] requires only p_z to be *small*. At present we have no unambiguous experimental detection of PO with $p_z > 0$.

Finally, we note that, in general, the approach of [15] would predict complex stationary phase points. Their complex part has been neglected in order for the theory to obtain PO's. Our work [17] indicates that the consistent failure of the PO formalism in the intermediate regime, despite the usual normal form corrections, may require that the usual PO picture be partly abandoned since complex *nonperiodic* contributions—as opposed to ghosts, which are complex PO's—may be essential.

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